1. Let $f(x) = x^3$.

(a) Evaluate $\frac{f(5+h) - f(5)}{h}$ for $h = 0.1$.

(b) What number does $\frac{f(5+h) - f(5)}{h}$ approach as h approaches zero?

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

2. Differentiate with respect to x

(a) $\sqrt{3-4x}$

(b) $e^{\sin x}$

Working:

Answers:

(a) ....
...

(b) ....
...

(Total 4 marks)

3. Differentiate with respect to x:

(a) $(x^2 + 1)^2$.

(b) $1n(3x - 1)$.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)

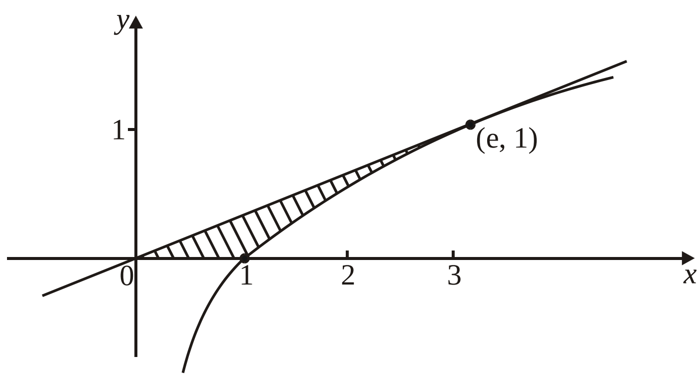
4. (a) Find the equation of the tangent line to the curve $y = \ln x$ at the point (e, 1), and verify that the origin is on this line.

(4)

(b) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x \ln x - x) = \ln x$.

(2)

(c) The diagram shows the region enclosed by the curve $y = \ln x$, the tangent line in part (a), and the line $y = 0$.



Use the result of part (b) to show that the area of this region is $\frac{1}{2}\mathrm{e} - 1$.

(4)
(Total 10 marks)

5. Let $f(x) = 1 + 3\cos(2x)$ for $0 \le x \le \pi$, and x is in radians.

(a) (i) Find $f'(x)$.

(ii) Find the values for x for which $f'(x) = 0$, giving your answers in terms of π.

(6)

The function $g(x)$ is defined as $g(x) = f(2x) - 1,\ 0 \le x \le \frac{\pi}{2}$.

(b) (i) The graph of f may be transformed to the graph of g by a stretch in the x-direction with scale factor $\frac{1}{2}$ followed by another transformation. Describe fully this other transformation.

(ii) Find the solution to the equation $g(x) = f(x)$

(4)
(Total 10 marks)

6. A ball is dropped vertically from a great height. Its velocity v is given by

$$v = 50 - 50e^{-0.2t},\ t \ge 0$$

where v is in metres per second and t is in seconds.

(a) Find the value of v when

(i) $t = 0$;

(ii) $t = 10$.

(2)

(b) (i) Find an expression for the acceleration, a, as a function of t.

(ii) What is the value of a when $t = 0$?

(3)

(c) (i) As t becomes large, what value does v approach?

(ii) As t becomes large, what value does a approach?

(iii) Explain the relationship between the answers to parts (i) and (ii).

(3)

(d) Let y metres be the distance fallen after t seconds.

(i) Show that $y = 50t + 250e^{-0.2t} + k$, where k is a constant.

(ii) Given that $y = 0$ when $t = 0$, find the value of k.

(iii) Find the time required to fall 250 m, giving your answer correct to **four** significant figures.

(7)
(Total 15 marks)

7. Let $f(x) = x^3 - 2x^2 - 1$.

(a) Find $f'(x)$.

(b) Find the gradient of the curve of $f(x)$ at the point $(2, -1)$.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

8. Differentiate each of the following with respect to x.

(a) $y = \sin 3x$

(1)

(b) $y = x \tan x$

(2)

(c) $y = \dfrac{\ln x}{x}$

(3)

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(Total 6 marks)

9. The population p of bacteria at time t is given by $p = 100e^{0.05t}$.

Calculate

(a) the value of p when $t = 0$;

(b) the rate of increase of the population when $t = 10$.

Working:

Answers:

(a) ……………………………………………..

(b) ……………………………………………..

(Total 6 marks)

10. The function f is given by $f(x) = 2\sin(5x - 3)$.

(a) Find $f''(x)$.

(b) Write down $\int f(x)\mathrm{d}x$.

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(Total 6 marks)

11. Let $f(x) = \dfrac{3x^2}{5x-1}$.

(a) Write down the **equation** of the vertical asymptote of $y = f(x)$.

(1)

(b) Find $f'(x)$. Give your answer in the form $\dfrac{ax^2 + bx}{(5x-1)^2}$ where a and $b \in \mathbb{Z}$.

(4)
(Total 5 marks)

12. Let $f(x) = 6\sqrt[3]{x^2}$. Find $f'(x)$.

Working:

Answer:

……………………………………………….......

(Total 6 marks)

13. Let $f(x) = \sqrt{x^3}$. Find

(a) $f'(x)$;

(b) $\int f(x)\mathrm{d}x$.

Working:

Answers:

(a) ……………………………………………………………
…

(b) ……………………………………………………………
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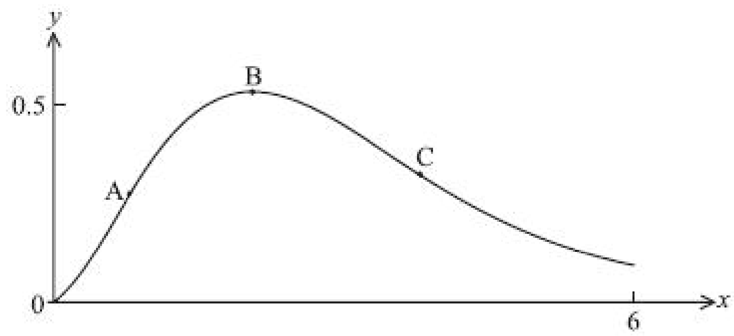
(Total 6 marks)

14. The diagram below shows the graph of $f(x) = x^2 e^{-x}$ for $0 \le x \le 6$. There are points of inflexion at A and C and there is a maximum at B.



(a) Using the product rule for differentiation, find $f'(x)$.

(b) Find the **exact** value of the ***y*-coordinate** of B.

(c) The second derivative of f is $f''(x) = (x^2 - 4x + 2)e^{-x}$. Use this result to find the **exact** value of the x-coordinate of C.

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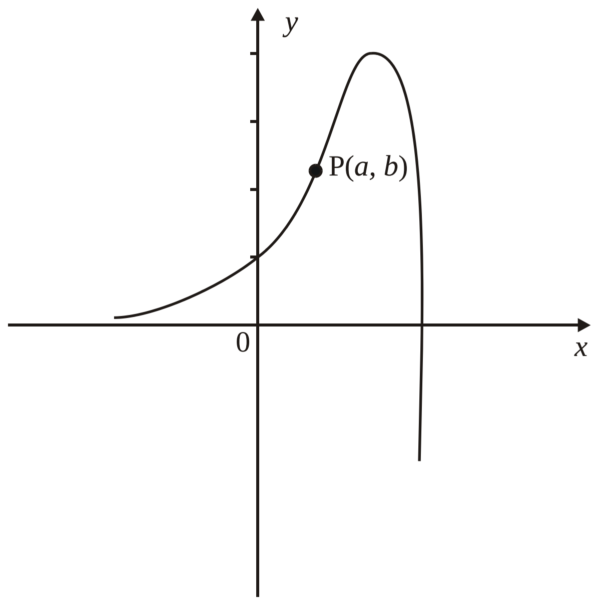
(Total 6 marks)

15. The diagram shows part of the graph of the curve with equation

$$y = e^{2x} \cos x.$$



(a) Show that $\frac{dy}{dx} = e^{2x}(2 \cos x - \sin x)$.

(2)

(b) Find $\frac{d^2y}{dx^2}$.

(4)

There is an inflexion point at P (a, b).

(c) Use the results from parts (a) and (b) to prove that:

(i) $\tan a = \frac{3}{4}$;

(3)

(ii) the gradient of the curve at P is e^{2a}.

(5)
(Total 14 marks)

16. Given the function $f(x) = x^2 - 3bx + (c + 2)$, determine the values of b and c such that $f(1) = 0$ and $f'(3) = 0$.

Working:

Answer:

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(Total 4 marks)

17. (a) Let $f(x) = e^{5x}$. Write down $f'(x)$.

(b) Let $g(x) = \sin 2x$. Write down $g'(x)$.

(c) Let $h(x) = e^{5x} \sin 2x$. Find $h'(x)$.

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(Total 6 marks)

18. Consider the function $f : x \mapsto 3x^2 - 5x + k$.

(a) Write down $f'(x)$.

The equation of the tangent to the graph of f at $x = p$ is $y = 7x - 9$. Find the value of

(b) p;

(c) k.

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(Total 6 marks)

19. Let $f(x) = (2x + 7)^3$ and $g(x) \cos^2(4x)$. Find

(a) $f'(x)$;

(b) $g'(x)$.

Working:

Answers:

(a) ....

(b) ....

(Total 6 marks)

20. The function f is given by

$$f(x) = 1 - \frac{2x}{1+x^2}$$

(a) (i) To display the graph of $y = f(x)$ for $-10 \le x \le 10$, a suitable interval for y, $a \le y \le b$ must be chosen. Suggest appropriate values for a and b.

(ii) Give the equation of the asymptote of the graph.

(3)

(b) Show that $f'(x) = \dfrac{2x^2 - 2}{(1+x^2)^2}$.

(4)

(c) **Use your answer to part (b)** to find the coordinates of the maximum point of the graph.

(3)

(d) (i) **Either** by inspection or by using an appropriate substitution, find

$$\int f(x)\,dx$$

(ii) **Hence** find the exact area of the region enclosed by the graph of f, the x-axis and the y-axis.

(8)
(Total 18 marks)

21. Given that $f(x) = (2x + 5)^3$ find

(a) $f'(x)$;

(b) $\int f(x)\mathrm{d}x$.

Working:

Answers:

(a) ....

(b) ....

(Total 4 marks)